

The AutoSkill® *Academy of Math*™ Correlation to NCTM Content Standards

Overview

In their comprehensive document, *Principles and Standards for School Mathematics* (2000), the National Council of Teacher's of Mathematics (NCTM) describe a set of ten mathematics standards. These standards serve as guidance for teachers in determining what students should be taught and what they should be able to do in the area of mathematics. These standards are ambitious, to be sure, and require a concerted effort among teachers, administrators and curriculum tools and resources.

The ten standards are divided into two groups. The NCTM **Content Standards** include: Number & Operations; Algebra; Geometry; Measurement; and Data Analysis & Probability. Each of these Content Standards includes a list of specific content objectives that explicitly describe the content that students should learn. The NCTM **Process Standards** include: Problem Solving; Reasoning & Proof; Communication; Connections; and Representations. These process standards describe ways of acquiring and utilizing mathematical content knowledge.

The AutoSkill *Academy of Math* program is designed to serve as a powerful curriculum enhancement tool by addressing the specific content objectives outlined within the NCTM Content Standards. Each of the *Academy of Math* program's 13,000-plus math problems can be aligned to at least one NCTM content objective.

Due to the nature of mathematics being a highly interconnected discipline, math problems, at various levels of complexity, often apply to more than a single content standard. Such is certainly the case with *Academy of Math* content. However, in an attempt to correlate to NCTM Content Standards, the problems within the *Academy of Math* have been correlated to one specific content objective.

The vast majority of the NCTM content standards are addressed by the *Academy of Math* content. Due to the nature of the program's design, some NCTM content standards are difficult to address, specifically those that require verbal reasoning or physical manipulation of mathematical tools. Arguably, these standards are better left to dynamic, teacher-driven math instruction in a classroom setting.

However, the NCTM content standards that address fundamental math skills, generally receive the most attention by the *Academy of Math* content. While the content is designed to foster mathematical proficiency, the greater emphasis placed on these fundamental skill standards indicates a focus on "computational fluency" within the *Academy of Math*.

NCTM Content Standards, Pre-K to 8	Academy of Math Content Areas, Levels 1 to 8	Number of questions in Levels 1 & 2 (2,904) (22%)	Number of questions in Levels 3 to 5 (4,934) (38%)	Number of questions in Levels 6 to 8 (5,210) (40%)	Total number of questions (13,048) (100%)
Number & Operations (Objectives include: Number sense & value; addition; subtraction; multiplication; division; fraction / decimals / percents; factorization; integers)	Numeration Addition Subtraction Multiplication Division Fraction	1,547 53%	2,694 54%	2,590 48.5%	6,831 52%
Algebra (Objectives include: Pattern recognition; properties of operations; symbolic notation; variables; equations; graphs; algebraic expressions; linear relationships)	Equations Graphing	544 19%	684 14%	989 20%	2,217 17%
Geometry (Objectives include: Two- and three-dimensional shapes; spatial relationships; symmetry; congruence & similarity; coordinates; angles; volume & area)	Geometry Graphing	237 8%	485 10%	511 10%	1,233 9.5%
Measurement (Objectives include: Length, volume, weight, area, time, standard units, metric and imperial systems; unit conversion; perimeter)	Measurement	346 12%	783 16%	750 14.5%	1,879 14.5%
Data Analysis & Probability (Objectives include: using data and describing what the data shows)	Graphing	230 8%	288 6%	370 7%	888 7%

Academy of Math and the NCTM Content Standards

Number & Operations

Traditionally, number sense has proved to be the cornerstone of the international mathematics curriculum. The NCTM maintains this tradition by ensuring that all standards, from pre-Kindergarten through grade 12, are strongly grounded in number.

In the NCTM standards, understanding number and operations, developing number sense, and acquiring fluency in arithmetic computations form the backbone of mathematics education.

According to the NCTM, “knowing basic number combinations – the single-digit addition and multiplication pairs and their counterparts for subtraction and division – is essential.” The NCTM continues by suggesting that, “equally essential is computational fluency – having and using

efficient and accurate methods for computing... Computational fluency should develop in tandem with understanding of the role and meaning of arithmetic operations in number systems.”

“The point is that students must become fluent in arithmetic computations – they must have efficient and accurate methods that are supported by an understanding of number and operations.”

The *Academy of Math* places an instructional emphasis on number and operations. Over 50% of the program’s content addresses this essential content standard. With specific content areas devoted to numeration, addition, subtraction, multiplication, division, and fractions, the *Academy of Math* ensures that students get the valuable experience required to develop efficient computational strategies. What is more, the program offers significant opportunities to practice working with numbers and operations within a variety of problem types allowing for the acquisition of computational fluency.

Algebra

The NCTM suggest that, “the ideas included in the Algebra standard constitute a major component of the school mathematics curriculum and help to unify it. Algebraic competence is important in adult life, both on the job and as preparation for post-secondary life. All students should learn algebra.”

It is important then, to view algebra as a strand in the curriculum from pre-Kindergarten on, so that students build a solid foundation of understanding and experiences as a preparation for more sophisticated work in algebra in the middle grades and high school.

With this in mind, AutoSkill incorporated mathematical instruction and practice in the *Academy of Math* that specifically addressed the NCTM algebra content objectives from pre-K to 8. The *Academy of Math* reserves 19% of the content from levels 1 and 2 to algebra-oriented questions. Approximately 17% of the overall program content is dedicated to algebra. As a result, students are able to develop and formalize their understanding of numeric and algebraic patterns, symbolic notation, variables and linear equations, among other areas. By working with algebra in a variety of forms and expressions, students training in the *Academy of Math* develop the foundation necessary for tackling the complex algebraic concepts they will encounter in high school and beyond.

Geometry

According to the NCTM, the notion of building understanding in geometry across the grades, from informal to more formal thinking, is consistent with the thinking of theorists and researchers. They further suggest that, “geometry is a natural place for the development of student’s reasoning and justification skills, culminating in work with proof in the secondary grades.”

Since geometric ideas are useful in representing and solving problems in other areas of mathematics and in real-world situations, geometry should be introduced to students in the early grades and should maintain a presence in the mathematics curriculum throughout school.

In an effort to foster geometric knowledge and understanding, the *Academy of Math* includes a robust geometry curriculum covering essential geometric concepts. With its instructional focus on two- and three-dimensional shapes and their attributes in the early levels, the *Academy of Math* lays the foundation for more complex geometry in the upper levels. By ensuring this foundation,

students are in a better position to grasp such concepts as coordinate geometry and transformations.

Measurement

The study of measurement is important in the mathematics curriculum from pre-Kindergarten through high school because of the practicality and pervasiveness of measurement in so many aspects of everyday life. Understanding what a measurable attribute is and becoming familiar with the units and processes that are used in measuring attributes is a major emphasis of this standard.

Just as the NCTM has placed a major emphasis on familiarity with standard units and measurable attributes, so too has the *Academy of Math*. Beginning in the early levels, and carrying forward to the upper levels, the *Academy of Math* offers substantial opportunity for students to become familiar with using and applying standard units when measuring a variety of attributes such as length, width, volume, area, weight, angles and time.

Since the Imperial system of measurement is still prevalent in the United States, students should learn both customary and metric systems and should know some rough equivalences between the metric and customary system. The study of these systems begins in elementary school, and students at this level should be able to carry out simple conversions within both systems.

The *Academy of Math* incorporates both the customary (Imperial) system and the metric system beginning in the early levels. Students become familiar with both systems, are given ample opportunity to convert from one unit to another within the same system and begin to understand rough equivalences across systems.

Data Analysis & Probability

The Data Analysis & Probability Standard recommends that students should formulate questions that can be answered using data and addresses what is involved in gathering and using data wisely. Students should learn how to collect data, organize their own or others' data, and display the data in graphs and charts that will be useful in answering their questions.

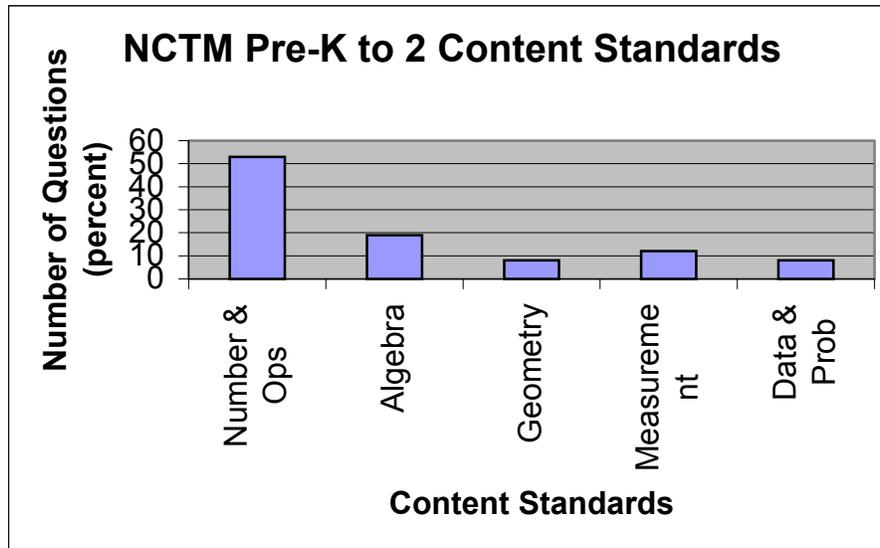
Because young children are naturally curious about their world, they often raise questions such as, How many? How much? What kind? or Which of these? Such questions often offer opportunities for beginning the study of data analysis and probability.

The *Academy of Math* lays the foundation for the study of data analysis and probability. The task of collecting, representing and analyzing data is a highly interactive process that requires students to be physically engaged. It is therefore important for students to have a firm understanding of the nature of graphs and the different ways that data can be represented and analyzed. The *Academy of Math* establishes this understanding through a variety of questions designed to familiarize students with graphing techniques.

Academy of Math and the NCTM Grade Bands

While the vast majority of the specific content objectives found within each Content Standard is addressed by the *Academy of Math* program, there is a clear instructional focus. These areas tend to be those that address fundamental math skills that require significant experience and practice in order to build fluency.

Academy of Math and the Pre-K to 2 Content Standards



Number & Operations

Areas of focus:

- Understand the effects of adding and subtracting whole numbers
- Understand various meanings of addition and subtraction of whole numbers and the relationship between the two.
- Understanding situations that entail multiplication and division, such as equal groupings of objects and sharing equally.
- Understand and represent commonly used fractions, such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Algebra

Areas of focus:

- Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another.
- Model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.
- Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notation.

Geometry

Areas of focus:

- Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes.
- Describe attributes and parts of two- and three-dimensional shapes.

Measurement

Areas of focus:

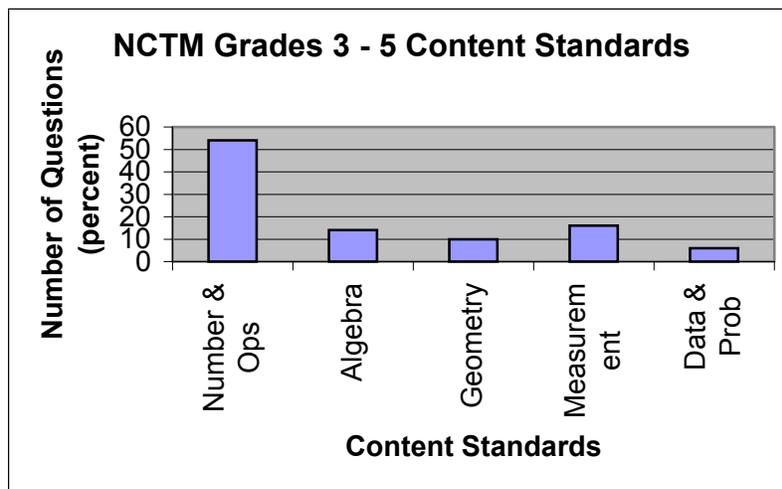
- Recognize the attributes of length, volume, weight, area, and time.
- Compare and order objects according to these attributes.
- Understand how to measure using nonstandard and standard units.

Data Analysis & Probability

Area of focus:

- Describe parts of the data and the set of data as a whole to determine what the data show.

Academy of Math and the Grade 3 –5 Content Standards



Number & Operations

Areas of focus:

- Understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals.
- Recognize and generate equivalent forms of commonly used fractions, decimals, and percents.
- Develop and use strategies to estimate the results of whole number computations and to judge the reasonableness of such results.
- Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.

Algebra

Areas of focus:

- Identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers.
- Represent the idea of a variable as an unknown quantity using as letter or a symbol.
- Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

Geometry

Areas of focus:

- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes.
- Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids.
- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.

Measurement

Areas of focus:

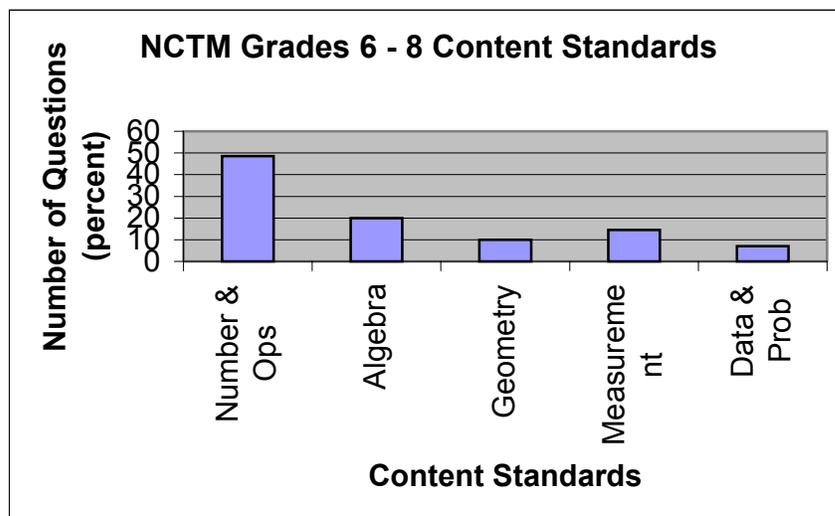
- Understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems.
- Carry out simple unit conversion, such as from centimeters to meters, within a system of measurement.

Data Analysis & Probability

Areas of focus:

- Represent data using tables and graphs such as line plots, bar graphs, and line graphs.
- Describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed.

Academy of Math and the 6 – 8 Content Standards



Number & Operations

Areas of focus:

- Develop and understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation.
- Use factors, multiples, prime factors, and relatively prime numbers to solve problems.
- Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.
- Develop and use strategies to estimate the results of rational number computations and judge the reasonableness of the results.

Algebra

Areas of focus:

- Recognize and generate equivalent forms for simple algebraic equations and solve linear equations.
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Geometry

Areas of focus:

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
- Use coordinate geometry to represent and examine properties of geometric shapes.

Measurement

Areas of focus:

- Understand relationships among units and convert from one unit to another within the same system.
- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.
- Develop and use formulas to determine the circumference of circles and the areas of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more complex shapes.

Data Analysis & Probability

Areas of focus:

- Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatter plots.
- Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots and scatter plots.